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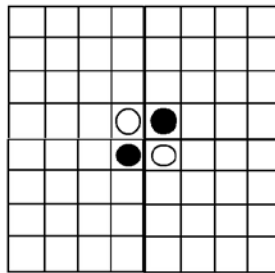
Advanced Game Theory

Professor David Housman

9 December 2010

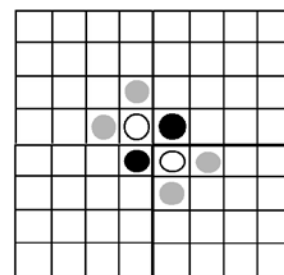
Othello

The game Othello, also known in by some people as Reversi, is a two player deterministic game. Player one uses black pieces and player two uses white pieces. Othello is usually played on a board made up of 64 spaces arranged in an 8x8 square. The game starts with four pieces in the center like so:

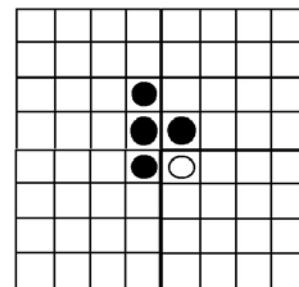


Then the players alternate turns, with player one, black, going first. A turn consists of a player laying one piece of his or her own color on an empty space and then flipping over pieces of the opposing player that are in a straight line, up, down or diagonal, between the piece just laid and an existing piece of the same color. A legal move is one that causes at

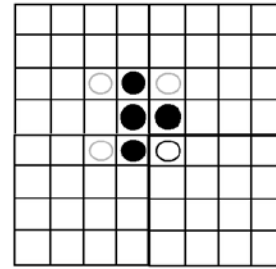
least one piece to be flipped. For example, for her opening move, player one can place a piece in any of the gray spaces, because any of those moves causes a white piece to be flipped over. If player



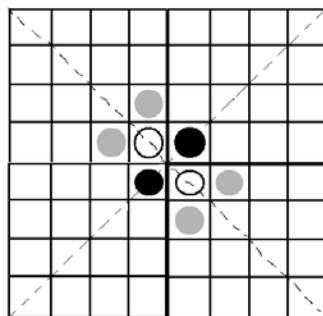
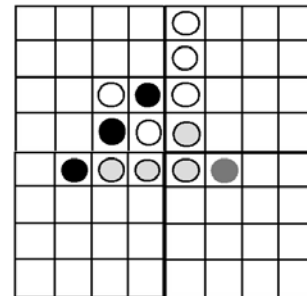
one places her piece in the third row from the top and the fourth column from the right, notated (row, column) as (3, 4), the white piece in (4, 4) will be flipped over, because it is between the new black piece and an old black piece.



Then player two takes a turn following the same rules, and could lay his piece in any of these spaces:



It is also possible for a player to flip over a number of pieces at once, in several directions. For instance, if player one, black, placed her piece in the dark gray space, all of player two's white pieces that are shaded light gray would be flipped over.

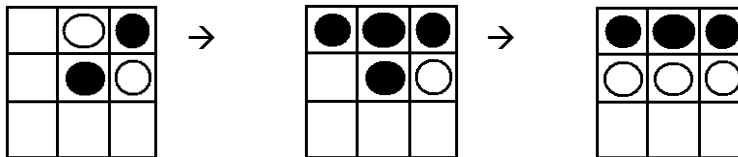


Any of player one's first moves are strategically the same by the properties of symmetry. No matter which move player one takes first, the board can be flipped over one or both of the lines of symmetry reach the other resulting boards.

If at any point the player whose turn it is is unable to execute a legal move, that player is skipped and the other player takes a turn if they can. The game is over when neither player has a legal move. The goal of the game is to have the most pieces at the end of the game. A player's score at the end of the game is the number of pieces they have. Degrees of winning are measured by the margin of difference between the two player's scores. A score will be denoted with square brackets [player one's score, player two's score], for example [24, 40] is a win for player two with a margin of difference of 16, which is better for player two than the score [29, 34] that only has a margin of difference of 5. Some basic strategies for the game include trying to place a

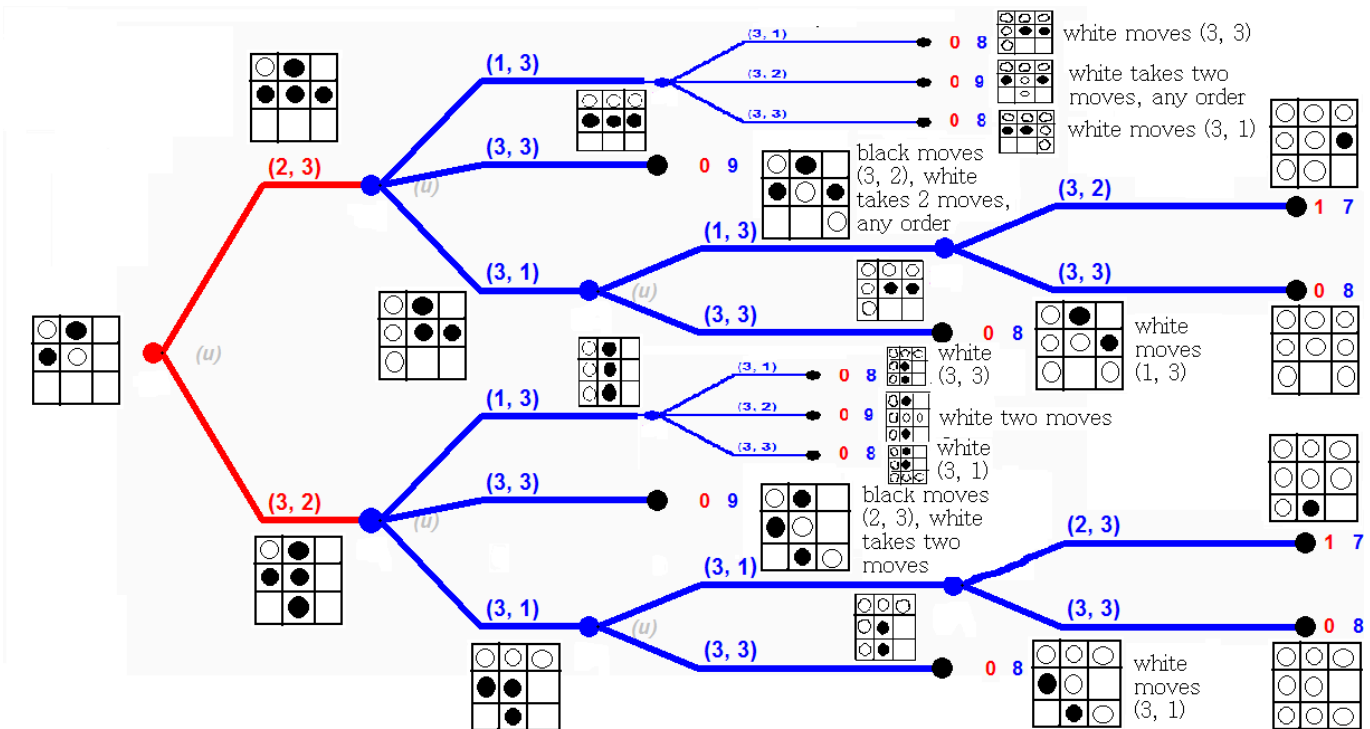
piece on an edge space or on a corner space because edge spaces are harder to flip over and once a piece is played in the corner it cannot be flipped.

Othello on an 8x8 board is a very complex game. There are 59 empty spaces after the first arbitrary move, and there are normally at least 3 legal moves for every turn, so the game would have about 3^{59} possible games, a number much too large for even a computer to work on. If Othello is simplified to a 3x3 board, the game becomes much simpler. In a 3x3 game, there are only 5 open spaces, so there much fewer games. A feature of a 3x3 game is that the initial set-up cannot be in the center, it must be in a corner. If the game starts in the top right corner, player one will always win. Player one can move either in space (1, 1) or (3, 3) which are the same move by symmetry, so we'll arbitrarily choose (1, 1). Then player two must choose (2, 1).



Then player two will never have any more moves. Player one can place in a corner first, and then only be able to play in the other corner, winning with a score of [8, 0], or she can place in (3, 2) first and then be able to place in both corners, winning [9, 0]. Since player one is the only player with real decisions, she will choose the preferable score or [9, 0] and place her piece in the center. A game starting in the lower left corner would be symmetrical to this game.

The 3x3 game starting in the upper right corner is more complicated, but there is still only one player, player 2, with non-arbitrary choices. The complete game tree is as follows, with player one in red and player two in blue. End games with no choices are left off of the tree and described in word and score instead.



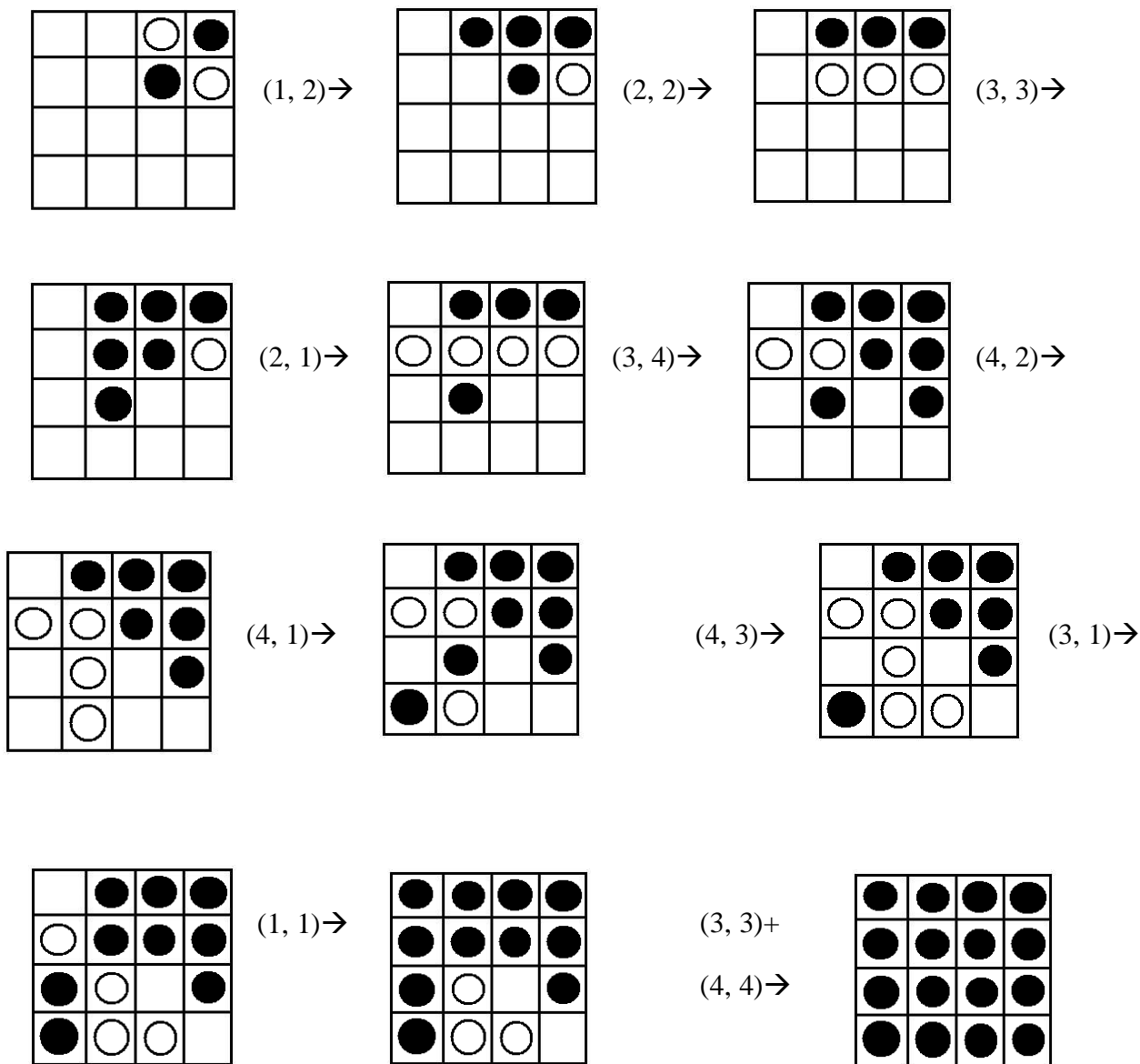
Since player two has the only real choices, he will choose a game that ends with a [0, 9] score.

He can accomplish this by choosing (3, 3) no matter what player one does. The game starting in the lower right corner would be symmetrical to this game.

The 4x4 Othello game has 11 empty spaces after the first arbitrary move, and therefore a potential of approximately 3^{11} moves, which is too large of a game to analyze by hand, but can be solved with a computer program. I wrote a computer program in Java that uses recursion to return the best move for a player when the board is at a certain position. The program uses a process similar to the building of a game tree and then doing backward induction to find the best move. If we were to ask the program for player one's best move at a certain position, the program would look at each of player one's legal moves and then ask player two what the score would be from his best move after that her move. The program would pick the best of these options and return it. Using the best moves for each player, we can construct the ideal game.

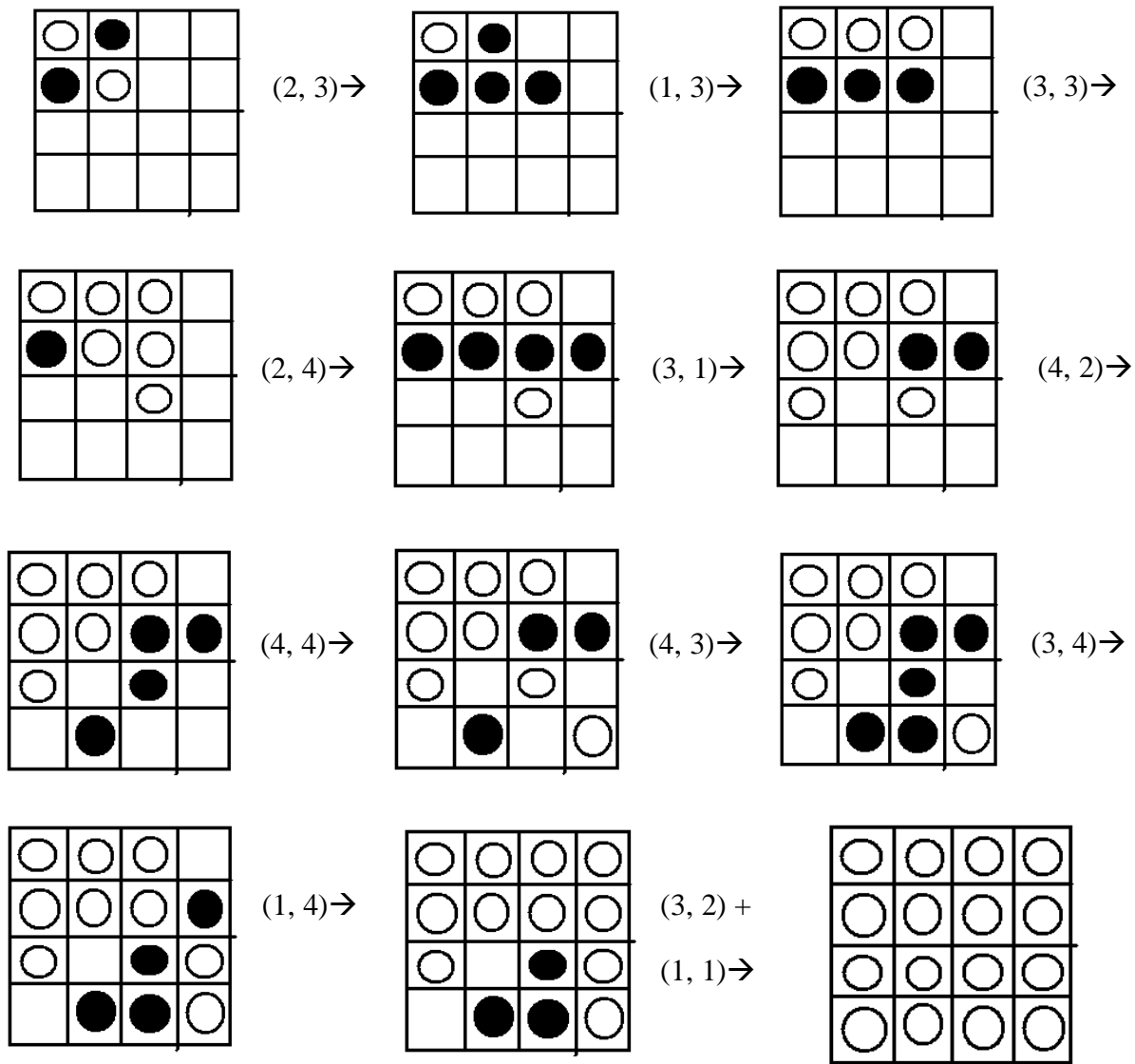
Sometimes two moves are equally good, so there may be more than one variation of the ideal game, but the program only returns one such game.

If the 4x4 game is started in the upper right corner, player one will win the ideal game with a score of [16, 0].



The game starting in the lower left corner is symmetrical to this game.

The 4x4 game starting in upper left corner is similar, except that player two will always win.



Player two can always win with a score of [0, 16]. The game starting in the lower right corner is symmetric to this game. In both the 3x3 and 4x4 games that start in the corner, the player whose piece is in the extreme of the corner will win the ideal game with the other player earning a score of zero. This pattern agrees with the strategy of place on corners.

The final game we will look at is the 4x4 game that starts in the center. The ideal game ends with player two winning with a score of [3, 11]. This is the game most like real Othello. It's interesting to note though, that while placing on corners is a general strategy, player one places on three corners but still loses.

